This question paper contains 2 printed pages. Roll No. ....

B.C.A. (Pt. - II) **Disc. Math.**

202/232

B.C.A. (Part - II) EXAMINATION, 2021

(Faculty of Science)

(Three - Year Scheme of 10+2+3 Pattern)

DISCRETE MATHEMATICS

Time Allowed : Three Hours Maximum Marks : 100

No supplementary answer-book will be given to any candidate. Hence the candidates should write their answers precisely in the main answer-book only.

All the parts of one question should be answered at one place in the answer-book. One complete question should not be answered at different places in the answer-book.

Write your roll number on question paper before start writing answers of questions.

PART - I : *(Very short answer) consists of **10** questions of **2** marks each. Maximum limit for each question is upto **40** words.*

PART - II : *(Short answer) consists of **5** questions of **4** marks each. Maximum limit for each question is upto **80** words.*

PART - III : *(Long answer) consists of **5** questions of **12** marks each with internal choice.*

PART - I

Attempt **all** parts of the question.

1. (a) Let $a \equiv b \pmod{x}$ and y be any integer then show that $a - y = b - y \pmod{x}$.
   (b) Expand $(1 + x)^5$ using Binomial theorem.
   (c) If $A \subseteq B$ then show that $A \oplus B = B - A$
   (d) Define equivalence relation.
   (e) Prove that $\sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$
   (f) Let $< B, +, \cdot, ', 0, 1 >$ be a Boolean algebra, then for all $a \in B$, prove that $a + a = a$.
   (g) Define simple graph.
   (h) Define product of two graphs.
   (i) Define Tree.
   (j) Define Spanning Tree.

PART - II

Attempt **all** the parts of the question.

2. (a) Solve $a_r = a_{r-1} + a_{r-2}$; $r \geq 2$, $a_0 = 0$, $a_1 = 1$.
   (b) If A, B, C and D are any four sets, then prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
   (c) If p and q are two statements then show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.
   (d) Prove that the numbers of edges in a simple graph with n vertices and k connected components $(k \geq 1)$ cannot exceed $\dfrac{(n-k)\,(n-k+1)}{2}$.
   (e) Prove that there is one and only path between every pair of distinct vertices in a tree.

PART - III

Attempt **all** questions by taking **any two** parts from each question.

3. (a) Prove that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n.

   (b) Find the Co-efficient x^r for the generating function $G(x) = \sum_{r=0}^{\infty} a_r x^r = \frac{x^2 - 5x + 3}{x^4 - 5x^2 + 4}$.

   (c) Solve the recurrence relation $a_r - 6\,a_{r-1} + 9\,a_{r-2} = r \cdot 3^r$

4. (a) How many integers are there between 1 and 1000 which are not divisible by 2, 3, 5 or 7 ?

   (b) Is the relation $R_1 = \{(a, b) | ab + 1 > 0 ; a, b \in R\}$ on the set R of real numbers, equivalence relation ? If not, explain.

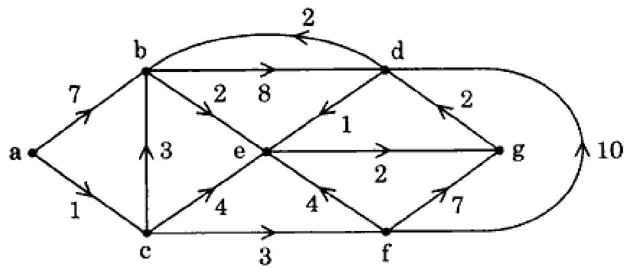
   (c) Prove that the inverse of a one-one onto function is one-one, onto.

5. (a) Prove by means of truth table, that $p \rightarrow (q \wedge r) \Leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$

   (b) In the Boolean algebra $< B, +, \cdot, ', 0, 1 >$, $\forall\, a \in B$, prove that $(a')' = a$.

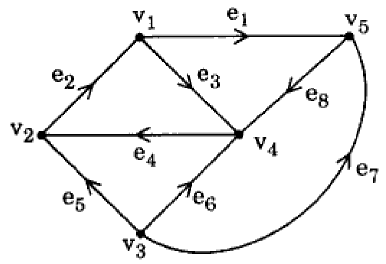
   (c) Prove that, no Boolean Algebra can have exactly three distinct elements.

6. (a) Find the shortest path between the vertices a and g in the following directed weighted graph.



   (b) If G is simple connected planer graph with n vertices and e edges (e > 2), then e ≤ 3n − 6.

   (c) Find the adjacency matrix and the incidence matrix of the following directed graph.



7. (a) If T is binary tree with n vertices and of height h, then prove that $h + 1 \le n \le 2^{h+1} - 1$.

   (b) Prove that a graph G is connected if and only if it has a spanning tree.

   (c) Discuss Kruskal's algorithm to find a minimal spanning tree for a weighted connected graph.

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