Basic Mathematics

B.C.A. (Part-I) Examination, 2017

[Time: Three Hours] **[Maximum Marks: 100]**

Part – I

1. Very Short Answer Type
   a) Define Invertible function.
   b) Define range of a function.
   c) Define transpose of a matrix.
   d) What difference between Eigen values and Eigen vectors.
   e) Write standard equation of a circle.
   f) Write Shridharacharya's formula.
   g) Define Dispersion.
   h) Write Relation between Mean, Mode, Median.
   i) Define permutation.
   j) Write Multiplication Law of probability.

Part – II

2. a) Prove that if f: $X \to Y$ and $g : Y \to Z$ are one-one function, then gof is also a one-one functions.

b) If $A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 2 & 5 \end{bmatrix}_{2\times3}$, $B = \begin{bmatrix} -1 & 0 & 3 \\ -2 & 5 & 1 \end{bmatrix}_{2\times3}$ find the matrix D such that $A + 2B - D = 0$.

c) Show that the point A(0, 1) , B(1, 4), C(4, 3) and D(3, 0) are the vertices of a square.

d) Calculate the median for the following frequency distribution.

x_i	1	2	3	4	5	6	7	8	9
f_i	8	10	11	16	20	25	15	9	6

e) Prove that

$$^nP_{n-1} = {}^nP_n.$$

Part – III

3. a) If $f: R \to R$ and $g: R \to R$ are the function where $f(x) = 2x + 3$ and $g(x) = x^2 - 1$ for all $x \in R$, then find $(f + g)(x)$, $(fg)(x)$, $(f+g)(-3)$ and $(fg)(5)$.

b) Define equal functions give an example of two functions that are equal.

OR

a) Show that the function $f: R \to R$, where $f(x) = x^3 + x$, for all $x \in R$ is a bijection.

b) If $f: R \to R$ where $f(x) = 2x - 3$ for all $x \in R$ then prove that f is bijective. Also find f^{-1}.

4. a) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A^T A = I_2$.

b) If $= \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, find $(A - 2I)(A - 3I)$.

OR

c) Prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x - y)(y - z)(z - x)$.

d) Solve the following system of equations by Cramer's rule:

$$x + y + z = 11$$
$$2x - 6y - z = 0$$
$$3x - 4y - 2z = 0.$$

5. a) Find the Locus of a point such that the sum of its distances from the points (2, 0) and (-2, 0) is always 6.

b) Derive the slope – Intercept form of the equation of straight line.

OR

a) Derive the normal form of the equation of straight line.

b) Prove that the following straight lines are concurrent.
$3x - 5y - 11 = 0, 5x + 3y - 7 = 0, x + 2y = 0.$

6. a) Calculate the mean and the standard deviation of first n natural numbers.

b) Calculate the mean, variance and standard deviation for the following frequency distribution:

Marks	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students(f_i)	3	6	13	15	14	5	4

OR

The following marks were obtained by class of students in Mathematics (out of 100). Compare the correlation coefficient for the above data. Find also the equations of the lines of regression.

Paper - I	45	55	56	58	60	65	68	70	75	80	85
Paper - II	56	50	48	60	62	64	65	70	74	82	90

7. a) Find the value of n if ${}^{7}P_{n} = 2.\ {}^{7}P_{n-2}$.

b) Let A and B be two events such that $P(\bar{A}) = {}^{2}/_{3}$ and $(A \cup B) = \frac{1}{2}$. Find $P(\bar{A} \cap B)$.

OR

a) To prove that $C(n, r) = C(n-1, r-1) + C(n-1, r)$, where $0 < r < n$.

b) Find the probability of getting a total of at least 6 in a simultaneous throw of three dice.